The order in the Cosmos

Homogeneity and isotropy in the Cosmos

Are the fundamental building blocks of the Universe evenly spread in space?

The Cosmological principle:

All point in space and all directions are identical.

What are the fundamental building building block of the Universe?

The Olbers' paradox: why the night sky is dark?

Kepler & Halley: could stars shine forever in an infinite Universe?

Heinrich Olbers (Jean Philippe de Cheseaux):

Assume an infinite Universe and stars as standard candles.

The apparent luminosity decreases as r^{-2}. The number of stars in a concentric shell with radius r and width dr is: $4\pi r^2 dr$

So that the total light at night should be: $\int_0^\infty r^{-2} 4\pi r^2 dr = \infty$

The Universe cannot be infinite, or something happens to the light on its way from far away towards us.

1929 - the emergence of observational Cosmology: the beginning of modern Cosmology. Hubble discovers that all galaxies move a way from us. Hubble law:

$$V=HR$$

Velocity of escape = (Hubble constant)(distance)

Age of the Universe = 1/H=15*10 ^{9} years

Hubble finds: H=500km sec^{-1} Mpc^{-1}

Present measured value:60-70km sec^{-1} Mpc^{-1}

The absolute horizon is at a distance of 5000Mpc

1pc=3.2 light years

On what scale is the Universe homogeneous?

How one checks the homogeneity?

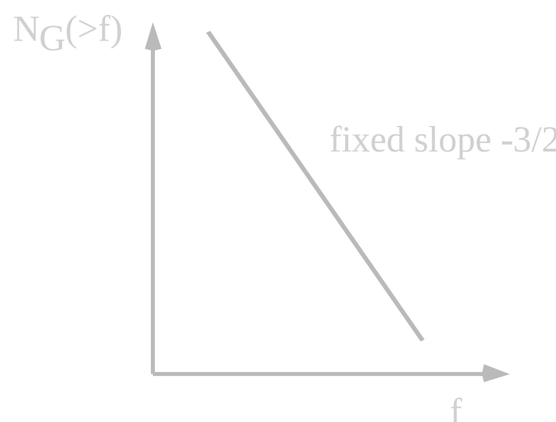
The assumption of an identical galaxy: all galaxies are identical

Standard candle assumption.

If the distance of a galaxy is s, the apparent intensity decreases as s^{-2}, but the number increases as s^3.

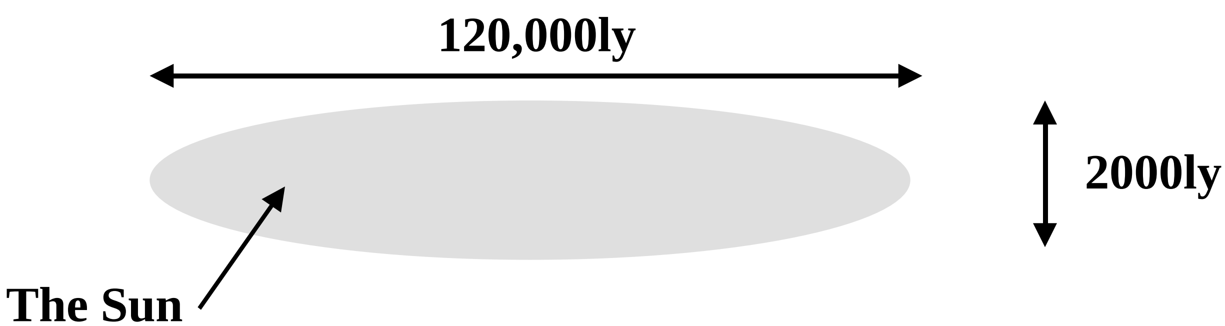
If the universe is homogeneous, the galaxies are evenly spread in space and the number of galaxies brighter than a given brightness f is proportional to $f^{-3/2}$.

Simple procedure:
Count galaxies and arrange
Them according to brightness



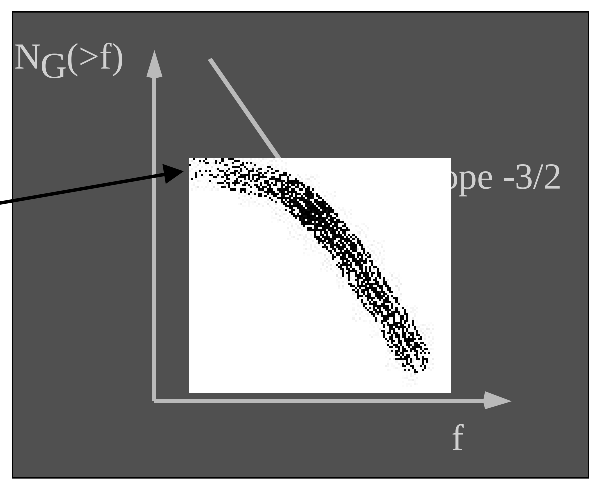
Few bits of history.

Kaptain picture (Beginning of 20th century)



No extra galactic object was known!

The edge of the galaxy



1923 Weyl predicts the Hubble law for sufficiently small distances.

1928 Robertson rediscovers the Weyl result.

1922 The theoretical implications of the Hubble law are derived by Friedman and Le Maitre (1927)

Einstein invents the cosmological constant to 'stabilize' the non steady universe.

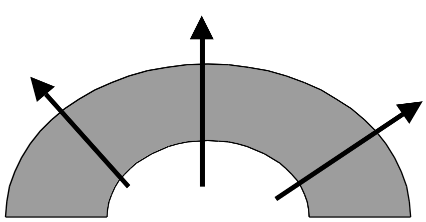
All the above takes place before Hubble's discovery

The fundamental building blocks of the Universe move away in all directions

A simple derivation of the Friedman equation

Consider an expanding shell in the Universe. Let the expansion velocity be v and the density inside the sphere ρ.

The kinetic energy is: $\frac{1}{2}v^2$



The gravitational energy is $-\frac{GM}{R}$

But: $M = \frac{4\pi}{3}\rho R^3$

And: $v = HR$ $\quad H = H(t)$

Conservation of energy: $$\frac{1}{2}H^2R^2 - \frac{4\pi}{3}G\rho R^2 = Const$$

According to the Big Bang hypothesis, this equation is valid for any shell

Re-scaling: $r = R(t)r_0$

R(t) is the scaling factor which expresses the change in the length between any two points in the Cosmos.

We can define today as: R(t=to day)=1

Define now: $$Const = 1/2kr_0^2$$

Where r_0 is arbitrary and $k = \pm 1, 0$

Substitute into the equation to find:

$$H^2 - \frac{8\pi}{3} G\rho = -\frac{k}{R^2}$$ **1st Friedman eq.**

Or:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho$$

The deceleration parameter is given by:

$$q(t) = -\left(\frac{\ddot{R}}{R}\right)\left(\frac{R}{\dot{R}}\right)^2 = -\frac{1}{H(t)^2}\frac{\ddot{R}}{R}$$

The present day value of q(t) is:

$$q_0 = q(t = now) = -\frac{1}{H_0^2}\frac{\ddot{R}_0}{R_0} = \frac{4\pi G\rho_0}{3H_0^2}$$

or

$$q_0 = -\frac{1}{H_0^2}\frac{\ddot{R}_0}{R_0} = -\frac{1}{(H_0 R_0)}\left(\frac{1}{H_0}\right)\ddot{R}_0$$

The red-shift of a galaxy is given by:

$$z = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{R(t_0)}{R(t)} - 1$$

$R(t_0)$ **The distance element at the photon emission time**

$R(t)$ **The distance element at the photon absorption time**

Solution of the first Friedman eq.

For k=0

$$\frac{R}{R_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}$$

For k=+1 we have only a parametric solution

$$t = \frac{q_0}{H_0} \frac{1}{(2q_0 - 1)^{3/2}} [\theta - \sin\theta]$$

The maximum radius is:

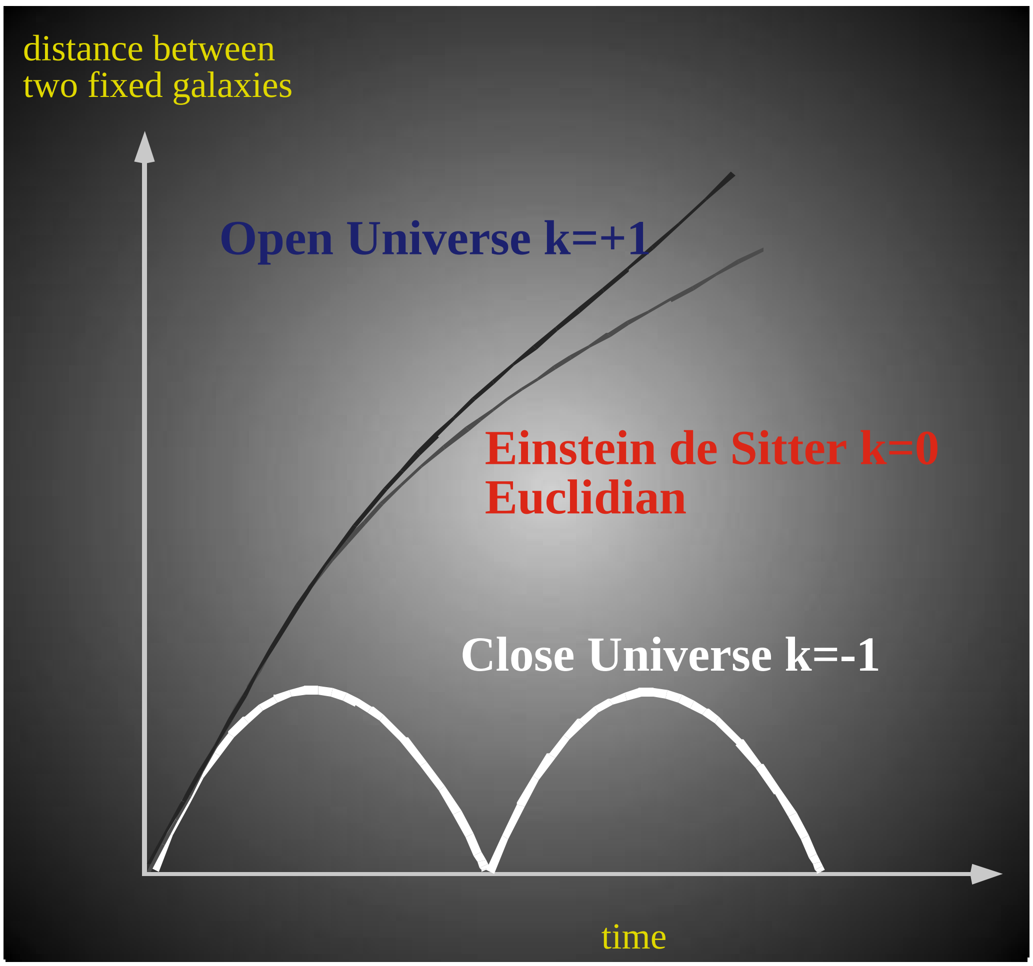
$$t_{\max} = \frac{1}{H_0} \frac{\pi q_0}{(2q_0 - 1)^{3/2}} \qquad R_{\max} = R_0 \frac{2q_0}{2q_0 - 1}$$

For k=-1

$$t = \frac{1}{H_0} \frac{q_0}{(2q_0 - 1)^{3/2}} (\sinh\phi - \phi)$$

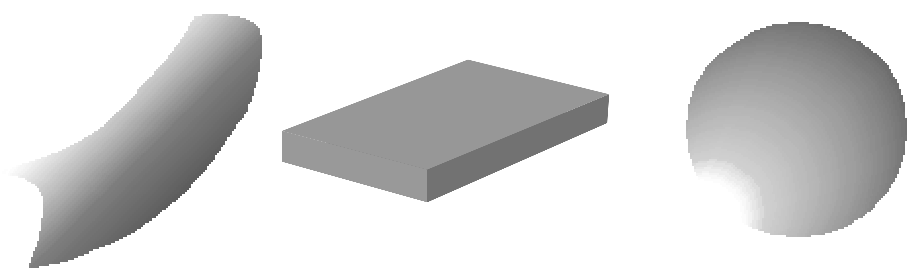
Again, only parametric form.

The three types of solutions to the Friedman equation



Possible solutions to the Friedman equation

curvature	$k > 0$	[redacted]	$k < 0$
geometry	hyperbolic	Euclidian	spherical elliptic
circle circumference	$> 2\pi R$	$= 2\pi R$	$< 2\pi R$
circle area	$> \pi R^2$	$= \pi R^2$	$< \pi R^2$



Let ρ be the total density of matter

If the expansion is adiabatic, then: $dE + PdV = 0$

Hence: $P = -\frac{dE}{dV}$

But $dV = \frac{4}{3}\pi R^3$ **&** $E = \rho V$

Consequently $P = -\frac{d(\rho V)}{dV} = -\frac{1}{3R^2}\frac{d(\rho R^3)}{dR}$

$$\frac{d}{dR}\left(\rho R^3\right) = -3PR^2$$ **2nd Friedman eq.**

The gas must have an equation of state. So the 3rd Friedman eq. is:

$$P = f(\rho, T)$$

For example, for a hot relativistic gas we have: $\rho \sim 3P$

The heat equation is:

$$\underbrace{dQ = Tds}_{\textit{Heat}} = \underbrace{d(\rho V)}_{\textit{internal energy}} + \underbrace{PdV}_{\textit{work}}$$

$$= d(\rho R^3) + PdR^3$$

For a gas of particles with rest mass m and number density n

$$\rho = mn + \frac{P}{\gamma - 1}$$

γ **is the adiabatic constant**

The solution of the Friedman eq. is:

The number of particles is conserved

In adiabatic expansion $dQ = 0$

The solution is then:

For positive density one gets negative pressure!

Einstein solution:

[illegible] cosmological constant

The revised Friedman equation is:

$$\left(\frac{\dot{R}}{R}\right)^2 = -k\frac{c^2}{R^2} + \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda$$

or:

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 = -k\frac{c^2}{R^2} + \frac{8\pi G}{3}P + c^2\Lambda$$

If the gas has no pressure (or it is negligible) the new equations reduce to the previous ones.

The modified equation with the cosmological constant behave as if there is repulsion between bodies. This repulsion balances the gravitational attraction.

The cosmological constant was introduced because the Friedman equations do not have a stationary solution.

So, before Hubble, Einstein modifies the governing equation of the Universe to obtain the supposedly stationary Universe.

The physical implication of the cosmological constant:

Suppose the cosmological term is the dominant one in the first equation. Then:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{c^2}{3}\Lambda$$

The solution is

$$R(t) = R_0 e^{\sqrt{\frac{c^2\Lambda}{3}}t}$$

Exponential expansion!

The critical density: Rewrite the Friedman equation for the density:

$$\rho = \frac{3H^2}{8\pi G} + \frac{3}{8\pi G}\frac{k}{R^2}$$

Define as the critical density (to day) $$\rho_c = \frac{3H^2}{8\pi G}$$

So that: $$\rho = \rho_c + \frac{3}{8\pi G}\frac{k}{R^2}$$

In a Euclidian Universe: $$\rho = \rho_c$$

$$\rho_c = 2\times 10^{-29} h^2 (gcm^{-3})$$

$$h = 100\ km\ \sec^{-1} Mpc^{-1}$$

Define: $\Omega(t) = \dfrac{\rho(t)}{\rho_c(t)}$ **then** $\dfrac{k}{R^2} = H^2(\Omega(t) - 1)$

or: $$\Omega(t) = 1 + \frac{k}{H^2 R^2}$$

Assume the cosmic gas to be relativistic so that:

$P = \dfrac{1}{3}\rho$ $\rho = aT^4$ **&** $RT = const$

Easy to prove

It can be shown that ***but we did not***

$$\left|\frac{\rho - \rho_c}{\rho}\right| = \left|1 - \frac{1}{\Omega}\right| = 1.88 \times 10^{-58} \left(\frac{m_p}{k_B T}\right)^2 k$$

What happens if the mass is not conserved?

Instead of

$$\frac{d}{dt}(nR^3) = 0$$

We have:

$$\frac{d}{dt}(nR^3) = (\alpha H^2)R^3$$

Formation volume

α is a free parameter which allows the expression α H ^{2} for the formation per unit volume

Also in this case the solution is an exponential expansion

Hence:

$$\frac{\rho}{\rho_c} = 1 \pm 1.88 \times 10^{-58} \left(\frac{m_p}{k_B T} \right)^2 k$$

Even when the temperature in the Universe was much higher than the mass of the proton, still the density is close to the critical density with fantastic accuracy! Changing k - the curvature - does not change the closness of the density to the critical density.

The Universe should be
very close to Euclidian

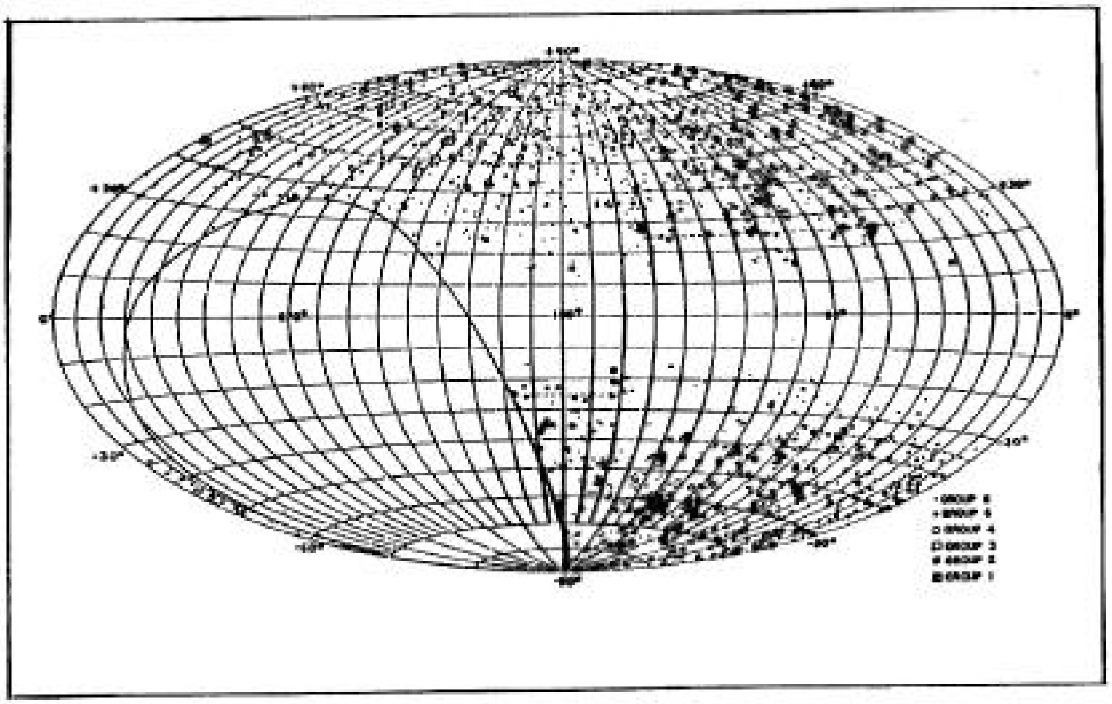


Fig 2A. The Distribution of Abell Clusters in Distance Groups 1 to 6
(Reproduced by kind permission of Prof. G. O. Abell)

The galaxy distribution found by Abel

Measuring the [illegible]

Does the Universe expand isotropically?

Let us write:

$$v^{\alpha} = H^{\alpha\beta} l^{\beta}$$

where H becomes a tensor.

$$H^{\alpha\beta} = H\delta_{\alpha\beta} + \Theta_{\alpha\beta} + \sigma_{\alpha\beta}$$

unti symmetric ***symmetric***

Observations indicate that close to a large cluster of galaxies

$$\left|\sigma_{\alpha\beta}\right| \approx 0.15H$$

The cluster of galaxies contains a huge amount of mass and disturbs the 'Hubble flow'

The best results today are:

$$\sigma_{11} = \quad 0.10H$$

$$\sigma_{22} = \ 0.03H$$

$$\sigma_{33} = -0.13H$$

The shear that results from non isotropic expansion is less than 10%.

Estimate of the mean density in the Universe

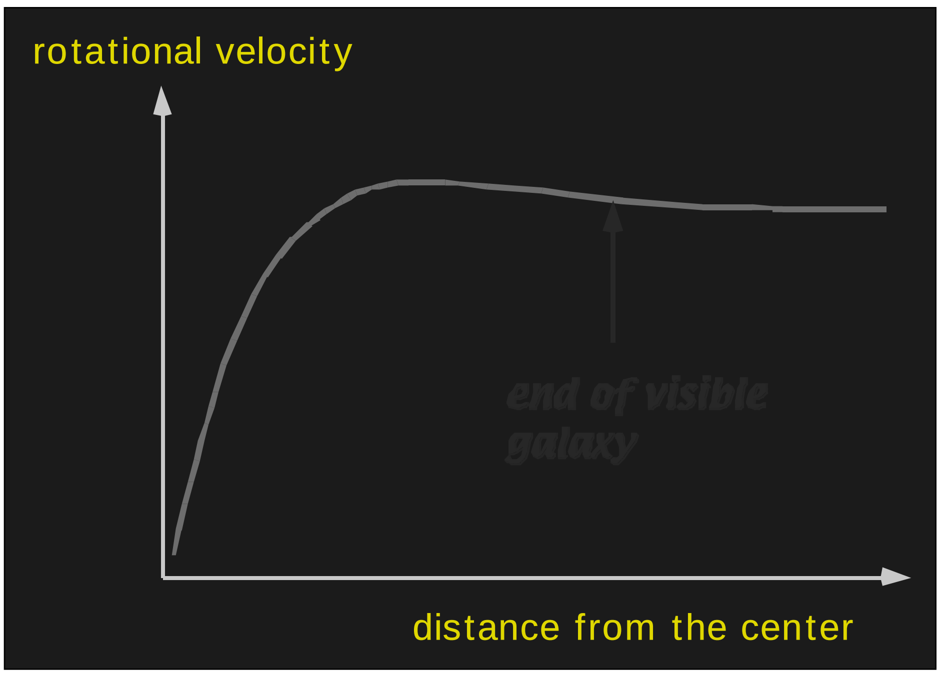
$$\rho(Galaxies) = L\left(\frac{m}{L}\right)$$

where

L is the total luminosity in the Universe per units volume contributed by the galaxies

$\frac{m}{L}$ **is the mean ratio of mass to luminosity for galaxies**

Observations of the rotational velocities of galaxies



One then solves the Poisson's equation to fins the density

A significant part of the galaxy is unobserved and does not contribute to the luminosity but contributes to the gravitational potential.

The value m/L contains a large uncertainty.

Similar methods applied to cluster of galaxies reveal again that a large fraction of the mass (up to 90%) is unobserved.

The ratio m/L increases with cosmic scale (Hoffman & Shaviv)

A significant part of the galaxy does not contribute to the total luminosity and hence is *unobserved!*

There is plenty of 'dark' matter which affects the gravitational potential of the galaxy but does not shine, or is very very dim.

It seems that the shining or visible matter is only 10% of the total matter in the Universe.

Most of the matter is not observed

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